Purchasing Power Protection with a Simple Trend-Following Strategy in Different Inflation Regimes

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Abstract

When faced with the question of how to build an inflation-protected portfolio, investors can choose assets with linear return profiles with respect to inflation, for example, real estate or inflation-linked bonds. Based on data from 1970 to 2011, the aim of this paper is to investigate which assets turn out to be most useful to achieve maximum real return under a risk constraint in different inflation environments. We are primarily concerned with inflation protection in a portfolio context. Assets considered in our analysis are the usual suspects, like equities, bonds, real estate and commodities. We also include inflation-protected bonds [based on the methodology by Kothari / Shanken (2004)]. As a representative for a non-linear return profile with respect to inflation, we propose a simple trend-following strategy shifting between commodities and bonds. For more risk-averse investors, real estate and inflation-protected bonds help to protect a portfolio against inflation. Irrespective of the level of risk aversion, trend-followers contribute to purchasing power protection.

Start of text

When faced with the question of how to build an inflation-protected portfolio, investors can choose assets with linear return profiles with respect to inflation, for example, real estate or inflation-linked bonds. When inflation picks up, the inflation hedges gain and when disinflation or deflation sets in, their values should decline. An undesired property of linear inflation hedges is that they tend to underperform in disinflationary or deflationary times.

As an alternative, trading-oriented assets generate non-linear return profiles. Take as an example a simple trend-following strategy which buys commodities during inflationary times and buys bonds during times of low inflation: If the trend-follower has skill, positive returns in both environments should result, leading to a convex return profile.[1]

Based on data from 1970 to 2011, the aim of this paper is to investigate which assets turn out to be most useful to achieve maximum real return under a risk constraint in different inflation environments. The 1970s is the last era of high inflation and of considerable interest for this study.

[1] Cf. Fung / Hsieh [1997, 2001], also Merton [1981].

We are primarily concerned with inflation protection in a portfolio context. Assets considered in our analysis are the usual suspects, like equities, bonds, real estate and commodities. We also include inflation-protected bonds [based on the methodology by Kothari / Shanken [2004]]. As a representative for a non-linear return profile with respect to inflation, we propose a simple trend-following strategy shifting between commodities and bonds.

After giving a brief summary on the findings in the literature on potential inflation hedges and how they are represented in our database, in section three we describe our simple trend-following model. Section four discusses empirical characteristics of the assets in our database and section five explains our approach to determine efficient portfolios in a lower partial moments framework. Section six summarises and concludes. The appendix describes how we proxy a long time series for inflation-protected bonds.

Assets for inflation protection

Ideal characteristics of an asset as an instrument to protect against inflation are:

1. High inflation beta to protect other parts of the portfolio.
2. Reliable, constant relationship between asset's performance and inflation.
3. Ideally, convex return profile, i.e. performs well in an inflationary as well as a disinflationary / deflationary environment. This helps to protect the portfolio in times of high uncertainty.
4. Protects against inflation over several time horizons.
5. Liquidity.

We do not list separately the requirement that the asset should help to diversify other assets in the portfolio. On the fifth point, evidence in the literature generally indicates that the longer the time horizon, the stronger [positive or negative] is the relationship between inflation and the returns of assets. In this paper, we look at inflation over a five year horizon.

Several studies conclude that stocks are an effective inflation hedge, but only in the very long run [ten years and more]: Based on the analysis of 200 years of data, Boudoukh / Richardson [1993] provide evidence that equities are a long run only inflation hedge. Ely / Robinson [1997] find that stocks maintain their values relative to CPI over long horizons. Ahmed and Cardinale [2005] document that stocks are an effective inflation hedge in the U.S. over a minimum of five years. They come to mixed results for UK and Germany. Lothian / McCarthy [2001] state that equities in OECD countries are a good inflation hedge, but it takes "an exceedingly long time for this to happen." Equities are represented in our empirical study by the MSCI USA total return index.

The Fisher equation can be used to understand the behaviour of bonds during inflation. It establishes a relationship between nominal yields [i_t], real yields [r_t], expected inflation [$E_{t-1}(\pi_t)$] and the inflation risk premium [irp_t]:

$$\boldsymbol{i_t = r_t + E_{t-1}(\pi_t) + irp_t} \tag{1}$$

The buyer of a bond secures the nominal coupons over the bond's lifetime. When inflation expections $E_{t-1}(\pi_t)$ rise, the nominal yield i_t increases and leads to losses on the bond. Nominal bonds, therefore, are little suited to protect against inflation. Empirically, correlations between bond returns and inflation are negative.[2] For our study, approximated total returns in percentages for five-year bonds are calculated with the formula:[3]

$$R_t = \frac{Y_{t-j}}{12} + D_{t-1}\left(Y_{t-j} - Y_t\right)$$

With R_t symbolising the total return, Y_t the yield of the bond at time t, and D_{t-1} the estimated duration of the bond. Yields for the five-year constant maturity bond were taken from the US Treasury's website [www.treasury.gov]. The same is true for yields of three-month T-bills which we converted into total returns.

Inflation-protected bonds establish an explicit link between statistical measures, like CPI, and the bond's performance. With a few months delay, the bond's principal is adjusted to reflect the increase of the CPI. They have existed in the US since 1997, in the UK since 1981. Japan, Sweden and the EMU countries France, Italy and Germany are amongst the issuing countries. US inflation-protected bonds have performed well in the early 2000s, when inflation expectations fell. Roll [2004] and Kothari / Shanken [2004] found favourable effects when inflation-linked bonds are added to portfolios. In recent years, the diversification properties of inflation-linked bonds may have become less pronounced, simply because inflation-protected bonds have become more correlated with nominal bonds.[4] Appendix A describes how we generated the synthetic time series for zero-coupon inflation-protected bonds based on the paper by Kothari / Shanken [2004].

Commodity returns are often based on futures contracts. Spot returns and futures returns are highly correlated, but the latter exceed spot returns.[5] Futures contracts on potential inflation hedges gold and crude oil only became available in 1975 or 1983. As we are very interested in the last high inflation era of the 1970s, we use spot commodity prices from UnctadStat going back to 1970 for our empirical investigation.

Oil is comprised in a large share of our daily goods [e.g., production, transport]. Its direct contribution to CPI growth as motor fuel is 5% and exerts a significant influence on CPI headline growth and its volatility. Combined with indirect effects oil is one of the most important drivers of CPI growth. Gold is widely perceived as an inflation hedge and serves as safe haven in uncertain times. It is difficult to assess the weight of cotton and soybeans, but rice, wheat, sugar, and coffee combined comprise a weight of ca. 2% of the CPI. In order to more easily represent these six soft commodities, we create an unweighted basket which we call SOFT. Each soft commodity accordingly has a share of 1/6 in this basket. Industrial Metals have a close relationship with economic growth, which in turn is tied to inflation. Similar to SOFT, we introduce a basket of industrial metals [INDM] as an unweighted basket with a weight of 20% each in aluminium, tin, zinc, copper and lead. Both commodity baskets SOFT and INDM are rebalanced annually. Costs of 0.5% per transaction are considered.

Real estate investments are often distinguished between securitised and unsecuritised property. Often-used indices for unsecuritised real estate come from NCREIF. One drawback, however, is that

[2] Cf. Gorton / Rouwenhorst [2006].
[3] Cf. Roll [2004].
[4] Cf. Brière / Signori [2009].
[5] Cf. Gorton / Rouwenhorst [2006].

data are only available from 1978. With our five year horizon we would leave out the high-inflation years of the early 1970s, which are of considerable interest for our study. We therefore turn to the prices of New Homes Sold in the United States, which are available from 1963. Prices for New Homes in the US were taken from the website of the US Census Bureau [www.census.gov]. This time series only includes capital appreciation and does not include the income from owning a property like, for example, rent. Unfortunately, there is no time series that provides the income component.[6] The sales prices for new homes in the US are based on actual transactions and are therefore not prone to appraisal error. As the homes considered are new, there is less variation in terms of quality of the properties comprising the indices than with other indices. Following Mizuno / Taber [2011], we calculate returns as the quarterly average of the monthly median price.

Huang / Hudson-Wilson [2007] investigate the relationship between inflation and capital appreciation on the one hand and of rental income on the other hand. They conclude that inflation hedging is by far caused by capital appreciation and to a much lesser degree by rental income.[7] The most effective inflation hedge is office, followed by apartment [reflected by the prices of New Homes Sold], warehouse and retail.

In contrast to unsecuritised real estate, eal estate investement trusts [REIT] do not seem to provide a useful inflation hedge.[8] Their returns correlate more with those of small cap equities than with inflation indicators. Therefore, REITs are not considered in this study.

Data for other potential inflation hedges, like farmland, timber and timberland, are either too short to include the high inflation period of the 1970s, are available only in annual periodicity or not publicly available at all. Those assets are excluded from our asset base.

Assets with non-linear return profile: Trend-followers

It can be expected that inflation or dis-/deflation materialises in long-term trends. In an inflationary environment, commodities should outperform bonds, whereas bonds should outperform commodities in a dis-/deflationary scenario. A simple trend-following strategy could be to switch to commodities under inflation, and switch to bonds when dis-/deflation prevails: Our simple trend-following model shifts into a long position in the commodity basket if its three-month momentum exceeds the three-month momentum of bonds, otherwise a long position in bonds is taken. The three-month momentum is calculated as:

$$\boldsymbol{M_t} = \frac{\boldsymbol{P_t}}{\frac{1}{3}\cdot\left(\sum_{i=0}^{2} \boldsymbol{P_{t-i}}\right)} \tag{2}$$

where P_t is the price of the asset at time t. Similar simple trading strategies are documented to be profitable when applied to stocks[9], foreign exchange[10] and commodities[11].

[6] Similar time series reflecting only price appreciation and excluding the income component were used, for example, by Froot [1995].
[7] Cf. Huang / Hudson-Wilson [2007].
[8] For example, cf. Chan et al. [1990], Gyourko / Linneman [1988], Froot [1995].
[9] Cf. Brock et al. [1992].
[10] Cf. Harris / Yilmaz [2009].
[11] Cf. Miffre / Rallis [2007].

Commodities are represented by a basket comprising 33% Gold, 33% Crude Oil and 33% Industrial Metals [INDM]. Industrial Metals are included because they show a close relationship with economic growth, which in turn is tied to inflation. Soft commodities only have a small share of the CPI, as discussed above. In addition, the relation of inflation beta vs. annualised return is more favourable for industrial metals than for soft commodities.[12] Bonds are represented by the five-year constant maturity bond index, as described above.

Our momentum algorithm resembles a rule-driven trading strategy used by Commodity Trading Advisors [CTA]. Edwards / Park [1996] analyse correlations of monthly CTA returns with the US CPI on a monthly and on an annual basis. They find a significant positive relationship for annual returns. A similar result is provided by Irwin / Brorsen [1985] who investigate the time of high inflation end of the 1970s / early 1980s.

Empirical Characteristics of Assets for Inflation Protection

Our database includes 166 quarterly returns from Q1, 1970, to Q2, 2011. Exhibit 1 gives descriptive statistics for the quarterly returns between January 1970 and June 2011. All returns are discrete returns in USD. Returns and descriptive statistics in Exhibit 1 and Exhibit 2 for the inflation-protected bonds are based on a five year constant maturity.

Exhibit 1: Data sources and descriptive statistics for the empirical investigation

Name of asset	Description	Source	mean	median	st.dev.	skew	kurt	JB	JB, p	AC1, p	AC4, p
US CPI	CPI for All Urban Consumers (CPI-U) 1982-84=100 (Unadjusted) - CUUR0000SA0	www.bls.gov	0.0428	0.0361	0.0174	1.12	4.48	45.10	0.0000	0.0000	0.0000
RICE	Rice, Thailand, white milled, 5% broken, nominal price quotes, FOB Bangkok	UnctadStat	0.0489	-0.0170	0.2438	1.19	6.69	137.15	0.0000	0.0000	0.0001
SUGA	Sugar, average of I.S.A. daily prices, FOB Caribbean ports (¢/lb.)	UnctadStat	0.1432	0.0331	0.4192	1.01	5.42	57.23	0.0000	0.0894	0.2968
SOYB	Soybean meal, in bulk, 44/45% protein, Hamburg FOB ex-mill	UnctadStat	0.1013	0.0085	0.3273	2.19	15.24	1488.66	0.0000	0.4400	0.1922
COFF	Coffee, other mild Arabicas, ex-dock New York (¢/lb.)	UnctadStat	0.0926	-0.0095	0.3831	1.33	5.55	68.56	0.0000	0.3692	0.4625
COTT	Cotton, U.S. Memphis/Eastern, Midd.1-3/32	UnctadStat	0.0541	0.0507	0.2374	0.39	7.69	135.30	0.0000	0.5695	0.8495
WEAT	Wheat, United States, n° 2 Hard Red Winter (ordinary), FOB Gulf	UnctadStat	0.0491	0.0283	0.2133	2.28	18.15	1296.92	0.0000	0.2840	0.0170
SOFT	Unweighted avg. of RICE, SUGA, SOYB, COFF, COTT, WEAT. Annual rebalancing	UnctadStat	0.1368	0.0860	0.1951	1.09	5.52	17.25	0.0002	0.0016	0.0001
LA	Aluminium high grade, London Metal Exchange, cash	UnctadStat	0.1012	0.0111	0.2696	0.18	5.83	35.55	0.0000	0.0427	0.2189
LP	Copper, grade A, electrolytic wire bars/cathodes, London Metal Exchange	UnctadStat	0.0893	0.0326	0.3038	0.47	4.97	45.25	0.0000	0.3441	0.4927
LD	Lead, London Metal Exchange, cash settlement	UnctadStat	0.0799	-0.0310	0.3054	0.40	3.58	10.53	0.0052	0.8801	0.1810
LX	Zinc, special high grade, London Metal Exchange, cash settlement	UnctadStat	0.1095	-0.0140	0.2906	0.72	5.49	74.63	0.0000	0.0001	0.0006
LT	Tin, London Metal Exchange, cash	UnctadStat	0.0636	0.0526	0.1954	-1.14	10.65	341.35	0.0000	0.1319	0.5775
INDM	Unweighted average of LA, LP, LD, LX, LT. Annual rebalancing	UnctadStat	0.0941	0.0405	0.2168	0.58	4.74	19.95	0.0000	0.1440	0.0482
CRUD	Avg. of UK Brent (light)/Dubai (med.)/Texas (heavy) equally weighted	UnctadStat	0.1975	0.0155	0.5689	6.89	69.33	28155.98	0.0000	0.6430	0.8767
NHSP	New Home Sales Price	www.census.gov	0.1018	0.0872	0.1400	-0.77	17.42	240.38	0.0000	0.1669	0.0020
TB3M	Three-Month T-Bills	www.treasury.gov	0.0572	0.0545	0.0168	-0.09	4.56	32.54	0.0000	0.0000	0.0000
5Y_TR	Yield of five-year constant maturity bond	www.treasury.gov	0.0801	0.0716	0.0719	0.70	5.67	76.27	0.0000	0.0962	0.1267
ZCIP	Zero-Coupon Inflation Protected Bonds	Synth. time series, see app. A	0.1112	0.1052	0.1099	-0.27	7.55	152.25	0.0000	0.5148	0.0097
GOLD	Gold	UnctadStat	0.1284	0.0231	0.2612	0.63	8.12	153.10	0.0000	0.1001	0.0000
SILV	Silver, 99.9%, Handy & Harman, New York (¢/troy ounce)	UnctadStat	0.1031	-0.0072	0.3505	2.27	14.22	858.32	0.0000	0.0559	0.0680
TF	Trend-following model	Comprised of INDM and 5Y_TR	0.2389	0.0920	0.3177	6.09	59.54	20493.94	0.0000	0.6072	0.2459
MSCI USA	MSCI total return equity index	Bloomberg [GDDUUS Index]	0.1208	0.1347	0.1877	-0.78	5.73	7.54	0.0231	0.5739	0.3956

Means, medians and standard deviations are annualised. The highest mean returns stem from the trend-following strategy [0.2389], crude oil [0.1975] and sugar [0.1432]. Most volatile are crude oil [0.5689], sugar [0.4192] and coffee [0.3831]. The trend-following strategy has a volatility of 0.3177. Least volatile are three-month T-Bills [0.0168], bonds [0.0719] and inflation-protected bonds [0.1099].

JB is the statistic of the Jarque-Bera test for a normal distribution, “JB, p” is its corresponding p-value with two degrees of freedom. According to this test, none of the 22 time series can be testified as normally distributed at a 5% level of significance. Skewness is positive for all but five assets, the most positive are from crude oil [6.89] and the trend-following strategy [6.09].Values for kurtosis partly exceed three by far.

[12] Cf. Erb /Harvey [2006].

Columns “AC1, p” and “AC4, p” represent the p-values of the Ljung-Box-Test with one or four lags. Apart from the CPI and real estate [NHSP], autocorrelations at lags one and four are present in three-month T-Bills, synthetic zero-coupon inflation-protected bonds as well as rice, zinc, gold and the industrial metals bucket [INDM].

Exhibit 2Exhibit 2 shows correlations of quarterly returns between January 1970 and June 2011 for the various assets.

Exhibit 2: Correlation matrix based on quarterly returns

	US CPI	TB3M	5Y_TR	ZCIP	MSCI USA	LA	LP	LD	LX	LT	INDM	GOLD	SILV	CRUD	RICE	SUGA	SOYB	COFF	COTT	WEAT	SOFT	NHSP	TF
US CPI	1.00																						
TB3M	0.50	1.00																					
5Y_TR	-0.16	0.35	1.00																				
ZCIP	0.01	0.02	0.44	1.00																			
MSCI USA	-0.20	-0.02	0.18	0.14	1.00																		
LA	0.01	-0.12	-0.18	-0.02	0.02	1.00																	
LP	-0.01	-0.01	-0.08	0.01	-0.02	0.40	1.00																
LD	0.04	-0.15	-0.24	-0.07	-0.03	0.35	0.41	1.00															
LX	0.04	-0.03	-0.22	-0.13	0.00	0.38	0.47	0.38	1.00														
LT	0.10	-0.02	-0.03	-0.01	-0.01	0.26	0.27	0.29	0.24	1.00													
INDM	0.05	-0.04	-0.16	-0.09	-0.01	0.63	0.66	0.63	0.69	0.55	1.00												
GOLD	0.20	-0.08	0.08	0.07	-0.10	0.06	0.16	0.10	0.12	0.17	0.20	1.00											
SILV	0.12	0.02	0.04	0.04	0.14	0.20	0.17	0.11	0.18	0.23	0.25	0.63	1.00										
CRUD	0.32	0.05	-0.08	0.00	-0.11	0.13	0.20	0.17	0.05	0.24	0.22	0.44	0.30	1.00									
RICE	0.13	0.01	-0.07	-0.09	-0.06	0.15	0.18	0.09	0.28	0.26	0.29	0.25	0.20	0.01	1.00								
SUGA	0.11	-0.02	0.03	0.05	-0.05	0.12	0.20	0.18	0.10	0.21	0.22	0.28	0.31	0.22	0.04	1.00							
SOYB	0.00	-0.05	-0.03	-0.02	-0.16	0.11	0.13	0.14	0.17	0.01	0.17	0.22	0.13	-0.03	0.22	0.15	1.00						
COFF	0.05	0.04	-0.03	-0.10	-0.02	0.18	0.22	0.12	0.04	0.12	0.19	0.05	0.04	0.06	0.03	-0.01	0.17	1.00					
COTT	0.11	-0.09	-0.18	-0.11	0.04	0.12	0.08	0.07	0.17	0.10	0.12	-0.02	-0.01	-0.01	0.16	0.07	0.05	-0.14	1.00				
WEAT	0.05	-0.04	-0.06	0.00	-0.01	0.09	0.08	0.13	0.26	0.17	0.21	-0.01	0.15	-0.03	0.27	0.19	0.15	0.02	0.16	1.00			
SOFT	0.11	-0.16	-0.11	0.00	0.03	0.40	0.34	0.41	0.24	0.27	0.47	0.31	0.27	0.12	0.37	0.53	0.49	0.38	0.22	0.39	1.00		
NHSP	-0.02	-0.12	-0.11	0.07	0.26	0.29	0.39	0.33	0.23	0.11	0.32	0.05	0.08	0.01	0.16	0.02	0.11	0.10	-0.02	0.04	0.46	1.00	
TF	0.30	0.09	0.01	0.04	-0.10	0.26	0.41	0.35	0.25	0.33	0.45	0.53	0.39	0.82	0.20	0.35	0.12	0.16	0.06	0.06	0.29	0.06	1.00

Bonds have negative correlations with the US CPI [-0.16], while inflation-protected bonds have slightly positive correlations to inflation [0.01]. The highest correlations to the US CPI exhibit three-month T-bills [0.50], crude oil [0.32], trend-followers [0.30], gold [0.20]. Equities [-0.20] correlate negatively with the US CPI. Nominal bonds and inflation-protected bonds correlate at 0.44.

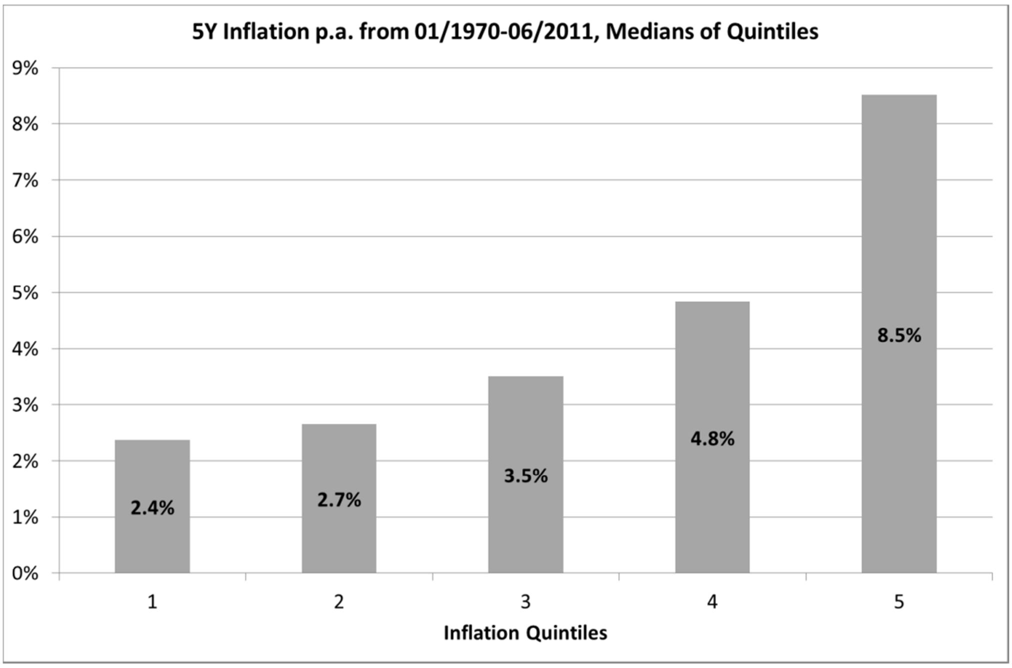
Correlations of US real estate [NHSP] with most assets are low in absolute terms [<= 0.2]. Exceptions are those with industrial metals, equities and the SOFT basket. Those of bonds with industrial metals are negative [-0.16], with gold slightly positive [0.08], and with crude oil negative [-0.08]. This might contribute to the high mean return of the trend-following strategy in Exhibit 1.

Based on the 166 quarterly returns, we create 147 overlapping five-year periods: the first five-year period comprises the 20 quarterly datapoints from Q1, 1970, to Q4, 1974. The second from Q2, 1970, to Q1, 1975 etc. The last five-year period stretches from Q3, 2006, to Q2, 2011.

Kothari / Shanken [2004] construct a time series of annual returns from holding a five-year bond for one year, resulting in a five-year constant maturity bond. For the purpose of our empirical study, it seems more realistic to us that an investor who wants to protect against inflation over a five-year horizon buys five-year inflation-protected bonds and holds them until maturity. His primary goal when buying those bonds is inflation protection and not duration management. At the beginning of the five-year period, we create a synthetic zero-coupon inflation-protected bond with a five year maturity. Each quarter, its principal is increased by the CPI change of the previous quarter, reflecting a one quarter adjustment lag. At maturity of the bond, its principal has risen by the cumulative increase of the CPI over the five-year period. Appendix A provides details of the calculation.

In order to investigate the inflation-protection properties of the assets in our database, we divide the data into quintiles according to five-year inflation. This gives 147 / 5 ≈ 29 five-year periods in each quintile. Quintile 1 comprises the five-year periods with the lowest inflation, mainly stemming from the 2000s. Quintile 2 contains mainly the early and late 1990s, quintile 3 mainly the 1980s, quintile 4 the early 1970s and late 1980s. Quintile 5 covers the five-year periods with the highest inflation, the 1970s. Exhibit 3**Error! Reference source not found.** gives the annualised medians of the five quintiles.

Exhibit 3: Medians of annual inflation, measured over five-year horizons



The value of 8.5% for the fifth quintile in Exhibit 3**Error! Reference source not found.** is the median annualised inflation rate of the 29 five-year horizons: Of the 29 five-year horizons with the highest inflation rates out of the 147 five-year periods, inflation in half of all cases fell below 8.5% and in 50% of those cases it turned out to be higher than 8.5%. The median of quintile 1 is only 2.4%.

Exhibit 4 shows the medians of the return standard deviations, sorted by inflation quintiles. Each standard deviation was calculated based on 20 quarterly returns for each five-year period, resulting in 147 standard deviations.

Exhibit 4: Medians of return standard deviations, sorted by inflation quintiles

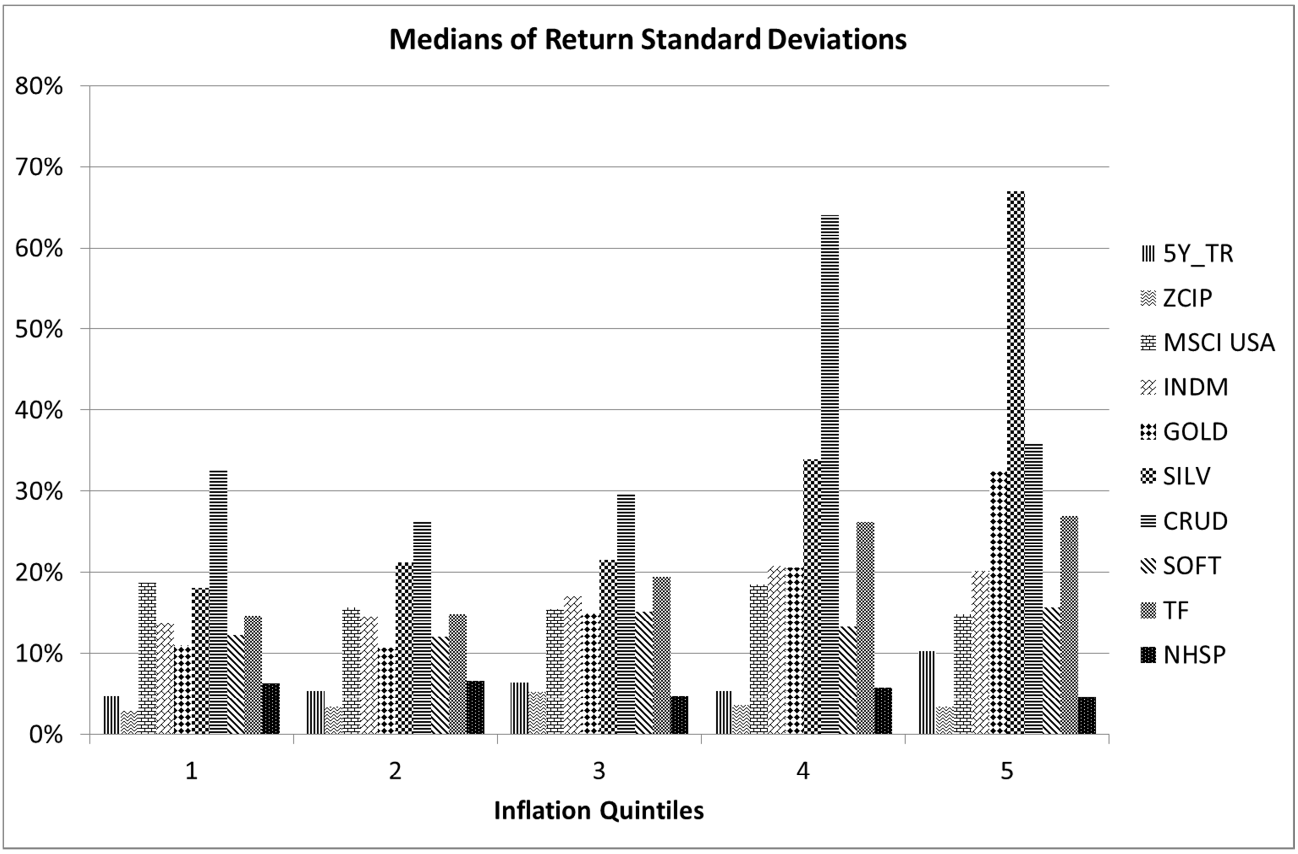
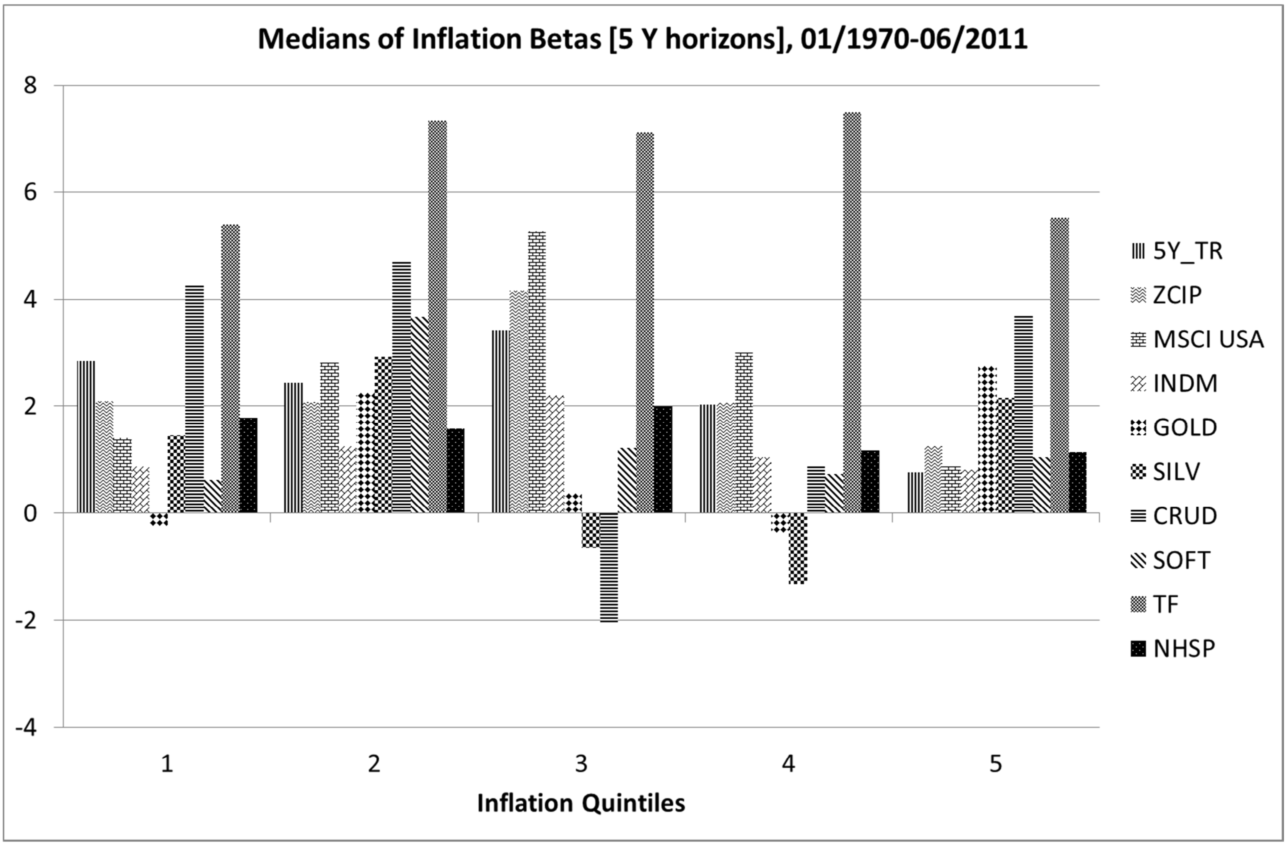


Exhibit 4Exhibit 4 displays that equities had about equal volatility in all inflation environments [medians 15% – 18%]. Similarly, volatility of real estate returns remains in a bandwidth of 5% to 7%, those of soft commodities between 12% and 16%. Industrial Metals show slightly higher volatilities, which increase from 13-17% in quintiles 1 to 3 to 20% in quintiles 4 and 5. Trend-Followers' volatilities rise from 14% in quintile 1 to 27% in quintile 5. This is due to shifting into more volatile commodities when inflation picks up and shifting into less volatile bonds when inflation is low. There is no regular pattern for crude oil, but its volatility is generally high [> 25%]. For gold and silver volatilities increase significantly with rising inflation.

The inflation betas in Exhibit 5Exhibit 5 show the medians of the five inflation quintiles.

Exhibit 5: Inflation betas sorted by quintiles



Inflation Beta is calculated as elasticity over the five-year horizon and not regression-based:

$$inflation\ beta = \frac{\frac{p_{i,t+5Y}}{p_{i,t}} - 1}{\frac{CPI_{t+5Y}}{CPI_t} - 1} \tag{3}$$

where $p_{i,t+5Y}$ is the price or index level of asset i at time t plus five years.

Equities [MSCI USA] exhibit low inflation beta with low inflation [quintile 1] and high inflation [quintile 5]. It is higher in quintiles 2 to 4, pointing to a “sweet spot of inflation” where equities perform well. Gold has historically realised inflation betas between -0.4 and +2.8, but there is no regular pattern of a relationship between gold and inflation. Similar applies to silver. Real Estate [NHSP] inflation betas are between 1.1 and 2.1 in all environments. The trend-following model produces high inflation betas in all environments [5.4 – 7.5]. Those of industrial metals [INDM] lie between 0.8 and 2.2.

Building an Inflation-Protected Portfolio

Optimisation problems are often formulated as maximising the portfolio return under consideration of a risk constraint. We define an individual’s risk constraint according to his current portfolio’s level of risk: If his benchmark portfolio comprises 10% cash and 90% bonds, his risk attitude is reflected in the return variation of this asset mix. We call this benchmark 1.

The next step is to investigate by how much the portfolio’s (real) return can be increased by changing the portfolio weights while leaving its risk constant. As most investors measure their returns and risk in nominal quantities, we define the target risk level in nominal returns. Given the fact that most time series in our data sample exhibit highly non-normal behaviour, the mean-variance framework would not be adequate for optimisation. Instead, we deploy the lower partial moments:[13]

$$LPM_n = \frac{1}{T}\sum_{t=1}^{T}[Max(0; r_{min} - r_{it})]^n \quad (4)$$

Where r_{it} is asset i's return at time t, r_{min} is the required minimum return [$r_{min} = 0\%$] and T equals the number of observations [T=20]. We use the lower partial moments with order n=2. Similar to the covariance matrix is the Co-Lower Partial Moments matrix. It can be approximated by the symmetrical $CLPM_2$ –matrix of the portfolio:[14]

$$sCLPM_{n,ij} = (sd_{n,i})(sd_{n,j})\rho_{ij} \quad (5)$$

where $sd_{n,i} = \left\{\frac{1}{T}\sum_{t=1}^{T}[Max(0; r_{min} - r_{it})]^n\right\}^{\frac{1}{n}}$ and ρ_{ij} is the correlation coefficient between assets i and j. Portfolio risk is defined as:

$$\boldsymbol{LPM_2^{Portfolio}} = \boldsymbol{w' \cdot sCLPM_2 \cdot w} \quad (6)$$

The letter **w** represents the vector of portfolio weights. Letters in bold symbolise matrices. Additional constraints are $w_i \geq 0$ and $\sum_{i=1}^{N} w_i = 1$.

For each of the 147 five-year periods, we calculate risk for benchmark 1 according to equation (6). Results of the optimisation are in Exhibit 6 [panels A to C]. All numbers are means of the quintiles, except for the inflation numbers, where medians are given. Returns and volatilities are annualised, LPM is per quarter:

Exhibit 6: Optimisation results for the investor with 10% Cash, 90% Bonds, 0% Equities [Benchmark 1]

[13] Cf. Bawa / Lindenberg [1977].
[14] Cf. Nawrocki [1991].

			Quintiles					
			all	1	2	3	4	5
	1	inflation p.a.	**3.5%**	2.4%	2.7%	3.5%	4.8%	8.5%
Panel A: optimal weights	2	TB3M	**10%**	10%	10%	10%	10%	10%
	3	5Y_TR	**49%**	73%	55%	66%	39%	15%
	4	ZCIP	**11%**	7%	10%	7%	27%	4%
	5	MSCI USA	**3%**	1%	8%	3%	4%	2%
	6	INDM	**2%**	0%	7%	2%	0%	0%
	7	GOLD	**1%**	3%	2%	0%	0%	0%
	8	SILV	**0%**	0%	0%	0%	0%	0%
	9	CRUD	**2%**	0%	1%	1%	0%	8%
	10	SOFT	**2%**	0%	0%	6%	1%	0%
	11	TF	**13%**	6%	7%	3%	3%	46%
	12	NHSP	**7%**	0%	0%	0%	16%	16%
Panel B: Optimised portfolio	13	return nom. pf_opt p.a.	**13.3%**	8.6%	11.3%	12.9%	13.0%	20.3%
	14	return real pf_opt p.a.	**8.7%**	6.2%	8.7%	9.4%	7.6%	11.6%
	15	pf opt vol p.a.	**7.3%**	4.0%	4.7%	6.0%	5.8%	15.8%
	16	pf opt LPM p.q.	**0.0128%**	0.0078%	0.0094%	0.0075%	0.0126%	0.0265%
Panel C: Benchmark 1	17	return nom. BM1 p.a.	**7.5%**	6.2%	5.5%	9.8%	9.1%	6.7%
	18	return real BM1 p.a.	**2.9%**	3.8%	2.8%	6.3%	3.7%	-1.9%
	19	BM1 vol p.a.	**5.6%**	4.5%	4.7%	5.7%	5.7%	7.5%
	20	BM1 LPM p.q.	**0.0130%**	0.0077%	0.0093%	0.0074%	0.0125%	0.0277%
	21	active real return p.a. [14-18]	**5.8%**	2.4%	5.9%	3.1%	3.9%	13.6%
Panel D: Inflation-protected portfolio 1	22	return nom. IPP1 p.a.	**11.0%**	7.9%	8.8%	11.6%	13.1%	13.4%
	23	return real IPP1 p.a.	**6.5%**	5.6%	6.1%	8.2%	7.8%	4.9%
	24	IPP1 vol p.a.	**4.9%**	3.0%	3.3%	4.3%	6.1%	7.9%
	25	IPP1 LPM p.q.	**0.0232%**	0.0173%	0.0159%	0.0205%	0.0290%	0.0332%
	26	real return IPP1-BM1 [23-18]	**3.6%**	1.8%	3.4%	1.9%	4.1%	6.8%

Bonds comprise the largest part of the optimised portfolios [row 3] with portions starting at 73% in the first quintile and declining to 15% in the fifth quintile. Inflation-protected bonds are represented in varying allocations from 4% to 27% without a clear pattern [row 4]. Form the first to third quintile the weights are between seven and ten percent. In the fourth it rises to 27% to decline to 4% in the highest inflation quintile. The reason for the at first glance counterintuitive small weight in quintile 5 is connected to the allocation of the trend-following strategy [row 11]: Trend-followers achieve their highest allocation in the high inflation quintile [46%], otherwise between 3% and 6%. Because of their high inflation beta, trend-followers generate more returns than the inflation-protected bonds in particular in periods of high inflation and are allocated higher weights despite their higher return variation.

Volatility of the benchmark is between 4.5% and 7.5% for all inflation quintiles [panel C, row 19]. It is highest in quintile 5. For the optimised portfolio [panel B, row 15], volatility increases with higher inflation from 4.0% [first quintile] to 15.8% [fifth quintile]. The volatility of the optimised portfolio is significantly higher than benchmark 1's. But the Lower Partial Moments for benchmark 1 and the optimised portfolio are at the same level [rows 20 and 16]. As the LPM-measure focuses on downside risk, the different volatilities between benchmark 1 and optimised portfolio are due to upside volatility. LPM is lowest for the third quintile and rises with increasing inflation for both benchmark 1 and the optimised portfolio [rows 16 and 20]. The highest LPM is realised in the fifth quintile. Real returns of the optimised portfolio are always higher than benchmark 1's [rows 14 and

18]. The active real return is +13.6% [row 21] and most pronounced in the fifth quintile. Even in the first quintile, the active real return is +2.4%.

Because of the risk constraint, equities are contained in only small shares between 1% and % [row 5]. Gold and other commodities as stand-alone investments achieve only small allocations of max. 7% [rows 6 to 10]. Commodities seem to contribute more effectively to inflation protection when included in a trend-following strategy. Real estate [row 12] has allocations of 16% in each of the two highest inflation quintiles and 0% otherwise. This proves that property contributes to portfolio performance in times of higher inflation, but is not well suited in times with lower inflation.

Now let's consider the investor is moving away from his 10% cash and 90% bonds allocation in order to achieve a better protection against inflation, but he still would like to accept only bond-like risk. He implements an allocation inspired by the average optimised allocation from Exhibit 6: 10% cash [TB3M], 50% bonds [5Y_TR], 5% equities [MSCI USA], 12.5% zero-coupon inflation-protected bonds [ZCIP], 12.5% trend-followers, 10% real estate [NHSP]. We call this portfolio Inflation-Protected Portfolio 1 [IPP1]. The weights are left unchanged over each five-year period. Panel D of Exhibit 6 shows the results for IPP1. The average annualised nominal return over all 147 five-year periods is 11.0% [row 22]. This contrasts with benchmark 1's 7.5% [row 17]. Mean nominal returns across all quintiles are higher for IPP1 than for benchmark 1. The same is true for real returns, which range from 4.9% to 8.2% for IPP1 [row 23] and from -1.9% to 6.3% for benchmark 1 [row 18]. Even in low-inflation quintile 1, IPP1 outperforms by 1.8%-points. Outperformance generally rises with increasing inflation [row 26]. Volatilities are at the same level as benchmark 1's across inflation quintiles [rows 19 and 24]. Downside risk, as measured by LPM, is higher for IPP1 [row 25] than for benchmark 1 [row 20].

We now repeat the exercise with a different risk attitude. Benchmark 2 comprises 10% cash, 45% bonds and 45% equities. Exhibit 7 gives the optimisation results.

Exhibit 7: Optimisation results for the investor with 10% Cash, 45% Bonds, 45% Equities [Benchmark 2]

			Quintiles					
			all	1	2	3	4	5
	1	inflation p.a.	**3.5%**	2.4%	2.7%	3.5%	4.8%	8.5%
Panel A: optimal weights	2	TB3M	**10%**	10%	10%	10%	10%	10%
	3	5Y_TR	**7%**	8%	8%	9%	12%	0%
	4	ZCIP	**1%**	0%	0%	4%	2%	1%
	5	MSCI USA	**18%**	26%	30%	21%	13%	3%
	6	INDM	**0%**	0%	1%	2%	0%	0%
	7	GOLD	**0%**	0%	0%	1%	0%	0%
	8	SILV	**1%**	0%	0%	0%	0%	3%
	9	CRUD	**1%**	2%	1%	2%	0%	1%
	10	SOFT	**0%**	0%	0%	2%	0%	0%
	11	TF	**59%**	54%	50%	49%	63%	81%
	12	NHSP	**0%**	0%	0%	0%	0%	0%
Panel B: Optimised portfolio	13	return nom. pf_opt p.a.	**20.8%**	20.1%	18.7%	18.4%	21.7%	29.1%
	14	return real pf_opt p.a.	**16.9%**	17.8%	16.1%	14.9%	17.3%	20.4%
	15	pf opt vol p.a.	**18.9%**	13.1%	13.1%	12.0%	23.5%	24.0%
	16	pf opt sCLPM p.q.	**0.0768%**	0.0887%	0.0603%	0.0629%	0.1998%	0.0544%
Panel C: Benchmark 2	17	return nom. BM2 p.a.	**9.3%**	4.5%	4.9%	12.6%	10.5%	7.0%
	18	return real BM2 p.a.	**4.6%**	2.3%	2.2%	8.9%	5.7%	-2.1%
	19	BM2 vol p.a.	**8.3%**	7.3%	6.5%	8.9%	9.5%	10.4%
	20	BM2 sCLPM p.q.	**0.0948%**	0.1296%	0.1297%	0.0825%	0.2341%	0.0552%
	21	active real return p.a. [14-18]	**12.3%**	15.5%	14.0%	6.0%	11.6%	22.5%
Panel D: Inflation-protected portfolio 2	22	return nom. IPP p.a.	**20.0%**	14.5%	18.2%	17.5%	22.5%	27.2%
	23	return real IPP p.a.	**15.5%**	12.2%	15.5%	14.0%	17.3%	18.7%
	24	IPP2 vol p.a.	**15.9%**	10.9%	10.8%	11.5%	21.3%	25.1%
	25	IPP2 sCLPM p.q.	**0.0887%**	0.0784%	0.0707%	0.0852%	0.1149%	0.0941%
	26	real return IPP2-BM2 [23-18]	**11.0%**	9.9%	13.4%	5.1%	11.6%	20.9%

Bonds are represented in only small amounts between 0% and 12% [panel A, row 3]. Even lower weights are assigned to inflation-protected bonds [row 4]: 0% to 4%. Equity allocations [MSCI USA] are on average 18%, decline in the two highest inflation quintiles 4 and 5 to 13% and 3% [row 5]. This adds to evidence in the literature that there are better inflation hedges than equities. Similar to the more conservative benchmark 1, long commodities including gold achieve only small weights between 0% and 2% [rows 6 to 10]. The dominant asset is the trend-following strategy [TF] with high allocations between 49 and 81% [row 11]. The highest weight is realised in the high inflation quintile 5. Real estate [row 12] weights are 0% in all quintiles.

Benchmark 2's volatilities are significantly below those of the optimised portfolio [rows 15 and 19], for example, in quintile 5, volatility of the optimised portfolio is 24.0%, whereas benchmark 2's is only 10.4%. Downside risk, however, as measured by the LPM, is lower or at the same level as for benchmark 2 [rows 16 and 20]. Active real returns are between 6.0% in quintile 3 and 22.5% p.a. for quintile 5 [row 21]. The average is 12.3%.

The Inflation-Protected Portfolio 2 [IPP2, panel D] is based on the average allocation of the optimised portfolios: 10% cash, 10% bonds, 20% equities and 60% trend-followers. IPP2 realises the best performances in the wing quintiles 1 and 5. This is true for nominal and real returns as well as for the relative performance vs. benchmark 2. In quintile 1, IPP outperforms benchmark 2 by 17% p.a., in quintile 5 by 25%.

Return volatilities are for all quintiles higher than benchmark 2's. The difference is most pronounced in quintile 5, where IPP2's volatility reaches 25.1% compared to benchmark 2's 10.4%. Again, most of this is upside volatility, as IPP2's downside [i.e. LPM] risk is lower or at the same level as benchmarks 2's. The exception is quintile 5.

Bearing in mind that a 60% allocation to trend-followers is not realistic for most investors, our results show that the addition of such a strategy can help to enhance portfolio returns in different inflation regimes.

Summary and Conclusion

Our study on inflation-protected portfolios divides a database from 1970 to 2011 into five different inflation regimes. We propose a simple trend-following strategy that shifts into commodities when inflation is on the rise and into bonds when dis- or deflation prevails. The trend-following strategy produces high inflation betas and works well in the more extreme cases of inflation, when the strategy mostly shifts into commodities. In the case of the low inflation regimes in our database, this includes the years of strongly negative equity markets which led to a flight to safe haven assets like government bonds. Portfolio optimisations show that long-only investments in commodities, like gold, oil or soft commodities only achieve very small allocations. In contrast, commodities bundled in a trend-following system can add value even in low-risk portfolios.

Real estate generates its best value in high inflation scenarios and in portfolios with lower risk tolerance. In low-inflation regimes, real estate was not considered in our portfolio optimisations. Inflation-protected bonds can be a useful addition to diversify and help to protect a part of the portfolio. Based on the four ideal characteristics of inflation hedging asset in section 2, Exhibit 8 gives a concluding overview of our empirical findings combined with those in the literature on characteristics of assets for inflation protection.

Exhibit 8: Summary of findings on ideal characteristics of an asset as inflation hedge

	inflation beta	reliable relationship to inflation	convexity with respect to inflation	Time horizon	Liquidity
Equities	low	no	no	long	high
Bonds	low	negative	no	medium	high
Real Estate	low	positive	no	medium	low
commodities [long-only]	high	no	no	medium	high
Gold	high	no	no	medium	high
Trend-Follower	high	positive	yes	medium	high
Inflation-Protected Bonds	low	positive	no	medium	medium

Appendix: Creating a Synthetic Time Series for Inflation-Protected Bonds

The procedure to build a long synthetic time series of total returns for zero-coupon inflation-protected bonds is documented in detail in Kothari / Shanken [2004]. Here, we only give a brief outline of the methodology, comment briefly on main points and show results.

While Kothari / Shanken [2004] use overlapping annual returns from monthly data, we deploy overlapping annual returns from quarterly data spanning June 1962 to June 2011. To proxy the return of a five-year bond, inflation expectations for the next five years are required. To this end, Kothari / Shanken run regressions on variables like interest rates and realised inflation. The estimated regression coefficients are used to forecast one-, two- and three-year ahead inflation. Four- and five-year forecasts are assumed to be identical to the three-year ones. We proceed in the same way. Exhibit 9 exhibits regression results based on our data in the left part and those from Kothari / Shanken [2004] in the right part.

Exhibit 9: Regression results from forecasting inflation according to the methodology of Kothari / Shanken [2004]

	June 1962-June 2011 [quarterly data]		Kothari/Shanken [2004], June 1953-December 2000 [monthly data]	
Variable	Coefficient	t-statistic	Coefficient	t-statistic
Forecast of one-year inflation [Eq. A.1]				
Intercept	0.02	3.38	0.98	2.22
Int_{t+1}	0.51	2.42	0.53	3.00
$Y_{t,t+5} - Int_{t+1}$	-0.35	-1.67	-0.21	-0.94
Inf_t	0.05	0.25	0.18	1.10
$Realbill_t$	-0.66	-2.98	-0.63	-3.18
Adjusted R^2	61.1%		71.0%	
Forecast of two-year inflation [Eq. A.2]				
Intercept	0.01	1.96	0.88	2.36
$Fint_{t+2} - Int_{t+1}$	0.96	2.54	0.80	3.17
Inf_t	-0.23	-3.33	-0.25	-3.91
$Realbill_t$	-0.17	-1.48	-0.13	-1.09
Adjusted R^2	20.6%		25.3%	
Forecast of three-year inflation [Eq. A.3]				
Intercept	0.00	-0.30	0.10	0.26
$Fint_{t+2} - Int_{t+1}$	1.12	1.75	0.83	2.25
Inf_t	-0.04	-0.37	-0.07	-0.93
$Realbill_t$	0.06	0.63	0.04	0.36
Adjusted R^2	5.3%		9.5%	

All estimated regression coefficients for the inflation forecasts have similar values and the same sign as the ones documented by Kothari / Shanken. All t-statistics were calculated with the Newey-West correction for autocorrelation. All returns for creating synthetic inflation-protected bonds are log-returns. Yields are interpolated linearly. T-bill yields and zero coupon yields were taken from the Federal Reserve's web page [www.federalreserve.gov].[15]

The starting point for the calculation of the synthetic zero-coupon inflation-protected bonds is equation (1):

$$\boldsymbol{i_t = r_t + E_{t-1}(\pi_t) + irp_t}$$

The inflation risk premium is assumed to be 0.[16] In a first step, inflation forecasts $E_{t-1}(\pi_t)$ need to be formulated. Step two uses those forecasts to calculate the real rate r_t. Step three computes nominal returns from the real rate r_t.

<u>Variables used:</u>

[15] See also Gurkaynak et al. [2006].

[16] A recent study, D'Amico et al. [2008], found that the inflation risk premium is a less important driver of nominal rates than prior papers.

Inf_t: Realised Inflation at time t. Expected inflation at time t for period t+k is represented by Inf_{t+k}.

Int_{t+1}: One year zero-coupon spot yield at time t.

$Realbill_t$: Sum of the realised real returns on three-month T-bills over the last twelve months.

$Fint_{t+k+1}$: Zero coupon forward rate from t+k to t+k+1:
$Fint_{t+k+1}$=$(t+k+1)\cdot Int_{t+k+1}-(t+k)\cdot Int_{t+k}$

Step 1: Estimating Expected inflation

Forecast for one-year inflation:

$$Inf_{t+1}=b_0+b_1Int_{t+1}+b_2\left(Y_{t,t+5}-Int_{t+1}\right)+b_3Inf_t+b_4Realbill_t+e_{t+1} \quad \text{(A.1)}$$

Forecast for the change of in inflation from year t+1 to year t+2 [change in expected two-year inflation vs. expected one-year inflation]:

$$Inf_{t+2}-Inf_{t+1}=b_0+b_1(Fint_{t+2}-Int_{t+1})+b_2Inf_t+b_3Realbill_t+e_{t+2} \quad \text{(A.2)}$$

Forecast for the change of in inflation from year t+2 to year t+3 [change in expected three-year inflation vs. expected two-year inflation]:

$$Inf_{t+3}-Inf_{t+2}=b_0+b_1(Fint_{t+3}-Fint_{t+2})+b_2Inf_t+b_3Realbill_t+e_{t+3} \quad \text{(A.3)}$$

Step 2: Estimating Real Interest Rates

Real returns are calculated by applying equation (1) and replacing the expected inflation rate $E_{t-1}(\pi_t)$ by its forecasted value Inf_{t+k}:

$$r_{t+k}=Int_{t+k}-Inf_{t+k} \quad \text{(A.4)}$$

Step 3: Calculating nominal returns

Every period, the principal of the inflation-protected bond is adjusted by the change in the CPI from period t-2 to t-1 [this reflects the adjustment lag]:

$$PV_{t+k}^{adj}=PV_{t+k-1}^{adj}\cdot\frac{CPI_{t+k-1}}{CPI_{t+k-2}} \quad \text{(A.5)}$$

The real rate, as calculated in step 2, is then used to determine the nominal price of the inflation-protected bond:

$$PV_{t+k}^{nominal}=PV_{t+k}^{adj}-(t+k)\cdot r_{t+k} \quad \text{(A.6)}$$

$PV_{t+k}^{nominal}$ can be used to calculate quarterly nominal discrete returns.

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